Graphical log-linear Rasch models

Rasch models with DIF and local-dependence

Testing Rasch models against GLLRMs

Model search

Testing GLLRMs

The DHP1 project

A subscale of the Diabetes Health Profile (DHP) measuring Disinhibited eating

A: DHP32 Do you wish there were not so many things to eat?

a) “Not at all”, b) “A little”, c) “A lot”, d) “Very much”

B: DHP34 How likely are you to eat something extra when you feel bored or fed up?

a) “Not at all likely”, b) “Not very likely”, c) “Quite likely”, d) “Very likely”

C: DHP36 When you start eating, how easy do you find it to stop?

a) “Very easy”, b) “Quite easy”, c) “Not very easy”, d) “Not at all easy”

D: DHP38 Is it difficult to keep your diet because you eat to cheer yourself up?

a) “Never”, b) “Sometimes”, c) “Usually”, d) “Always”

E: DHP39 Do you have problems keeping to your diet because you find it hard saying no to food you like?

a) “Never”, b) “Sometimes”, c) “Usually”, d) “Always”

DHP1 project also includes information on sex and age.

The Rasch model is rejected

	CLR	df	p
scoregroups	27.2	14	0.018
F: SEX	23.7	14	0.050
G: AGE	42.4	42	0.454

Tests of DIF

A & F:	lr =	1.47	df =	3	p =	0.6890
B & F:	lr =	3.19	df =	3	p =	0.3637
C & F:	lr =	12.72	df =	3	p =	0.0053
D & F:	lr =	5.47	df =	3	p =	0.1406
E & F:	lr =	4.93	df =	3	p =	0.1773
A & G:	lr =	9.71	df =	9	p =	0.3745
B & G:	lr =	5.68	df =	9	p =	0.7710
C & G:	lr =	13.08	df =	9	p =	0.1592
D & G:	lr =	9.91	df =	9	p =	0.9470
E & G:	lr =	9.91	df =	9	p =	0.3577

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| |
| Item restscore association |
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Item-restscore gamma

Item		observed	expected	sd	p
A -	DHP32	0.296	0.418	0.068	0.07447
B -	DHP34	0.494	0.460	0.061	0.56800
C -	DHP36	0.335	0.439	0.063	0.09764
D -	DHP38	0.681	0.422	0.073	0.00035
E -	DHP39	0.514	0.461	0.070	0.45254

Critical levels adjusted by the Benjamini-Hochberg procedure:
* < 5 % FDR, ** < 1 % FDR, *** = FDR < 0.1 % FDR

The Analysis of DIF

```
Analysis of DIF relative to F: AGE
Scale : # - RawScore

     Item   X²   df asymp exact  gamma asymp exact nsim
-------------------------------------------------------
A:   DHP32  86.0 63 0.029 0.060   0.05 0.683 0.689 1000
B:   DHP34  58.6 56 0.379 0.476  -0.12 0.299 0.286   21
C:   DHP36 101.4 66 0.003 0.012   0.14 0.233 0.212 1000  *
D:   DHP38  40.2 43 0.591 0.786  -0.22 0.113 0.155  103
E:   DHP39  55.5 51 0.309 0.473   0.21 0.120 0.122  222

Analysis of DIF relative to G: SEX
Scale : # - RawScore

     Item   X²   df asymp exact  gamma asymp exact nsim
-------------------------------------------------------
A:   DHP32  26.6 26 0.429 0.667   0.07 0.595 0.571   21
B:   DHP34  18.1 21 0.643 0.750   0.21 0.130 0.208   48
C:   DHP36  28.0 26 0.360 0.498  -0.32 0.012 0.019 1000  -
D:   DHP38  24.4 18 0.143 0.223   0.34 0.039 0.053 1000
E:   DHP39  27.1 21 0.168 0.235  -0.03 0.826 0.853   34
```

Evidence of DIF of C relative to G

Local dependence is also rejected

			A	B	C	D	E
A	DHP32	Gamma		0.149	-0.183	0.203	-0.157
		p		0.286	0.119	0.208	0.264
B	DHP34	Gamma	-0.055		-0.200	0.597	-0.122
		p	0.667		0.074	0.000	0.476
C	DHP36	Gamma	-0.204	0.060		-0.117	0.169
		p	0.062	0.630		0.286	0.194
D	DHP38	Gamma	-0.097	0.332	-0.530		0.260
		p	0.381	0.022	0.000		0.096
E	DHP39	Gamma	-0.200	-0.079	-0.052	0.480	
		p	0.134	0.667	0.711	0.003	

Graphical log-linear Rasch models

Too strong item - rest-score correlation happens when items are positively locally dependent

Instead of rejecting the Rasch model or elimination misfitting items and items that function differently in different groups we should try a model where local dependence and DIF is permitted.

Graphical log-linear Rasch models is one option

Graphical log-linear Rasch models with DIF

The Rasch model

$$\Pr(A=a,B=b,C=c,D=d,E=e \mid \theta,F=f,G=g) = \frac{\exp\left(r\theta+\sigma_a^A+\sigma_b^B+\sigma_c^C+\sigma_d^D+\sigma_e^E \mid \theta+F=f,G=g\right)}{H(\theta)}$$

$$r = a+b+c+d+e$$

The log-linear Rasch model with DIF of C relative to G adds an interaction parameter between C and G to the Rasch model

$$\Pr(A=a,B=b,C=c,D=d,E=e \mid \theta,F=f,G=g) = \frac{\exp\left(r\theta+\sigma_a^A+\sigma_b^B+\sigma_c^C+\sigma_d^D+\sigma_e^E+\varphi_{cg}^{CG} \mid \theta+F=f,G=g\right)}{H(\theta)}$$

φ_{cg}^{CG} **does not depend on θ**

R = A+B+C+D+E is sufficient for θ

The CLR test of no difference between the distribution of C in different groups defined by F is a test that $\varphi_{cg}^{CG} = 0$ **for all values of c and g.**

Graphical log-linear Rasch models with local dependence

The Rasch model

$$\Pr(A=a,B=b,C=c,D=d,E=e \mid \theta,F=f,G=g) = \frac{\mathrm{Exp}\left(r\theta+\sigma_a^A+\sigma_b^B+\sigma_c^C+\sigma_d^D+\sigma_e^E \mid \theta+F=f,G=g\right)}{H(\theta)}$$

The log-linear Rasch model with local dependence between D & E adds an interaction parameter between D and E to the Rasch model

$$\Pr(A=a,B=b,C=c,D=d,E=e \mid \theta,F=f,G=g) = \frac{\mathrm{Exp}\left(r\theta+\sigma_a^A+\sigma_b^B+\sigma_c^C+\sigma_d^D+\sigma_e^E+\lambda_{de}^{DE} \mid \theta+F=f,G=g\right)}{H(\theta)}$$

λ_{de}^{DE} **does not depend on θ**

R = A+B+C+D+E is sufficient for θ

To test that D & E are locally independent we calculate a CLR test of the hypothesis that $\lambda_{de}^{DE} = 0$ **for all values of d and e.**

Tests of local dependence between DHP1 items

```
  Check assumptions of local independence

A & B:    lr =    6.22   df =   9   p = 0.7182
A & C:    lr =   15.87   df =   9   p = 0.0696
A & D:    lr =   17.57   df =   9   p = 0.0405
A & E:    lr =   14.69   df =   9   p = 0.0999
B & C:    lr =   19.82   df =   9   p = 0.0190
B & D:    lr =   41.76   df =   9   p = 0.0000
B & E:    lr =    5.61   df =   9   p = 0.7780
C & D:    lr =    4.39   df =   9   p = 0.8839
C & E:    lr =    6.10   df =   9   p = 0.7295
D & E:    lr =   38.09   df =   9   p = 0.0000

  Benjamini & Hochberg rejects at 0.01000

     Suggested additions to the model:
            LD:              BD DE
```

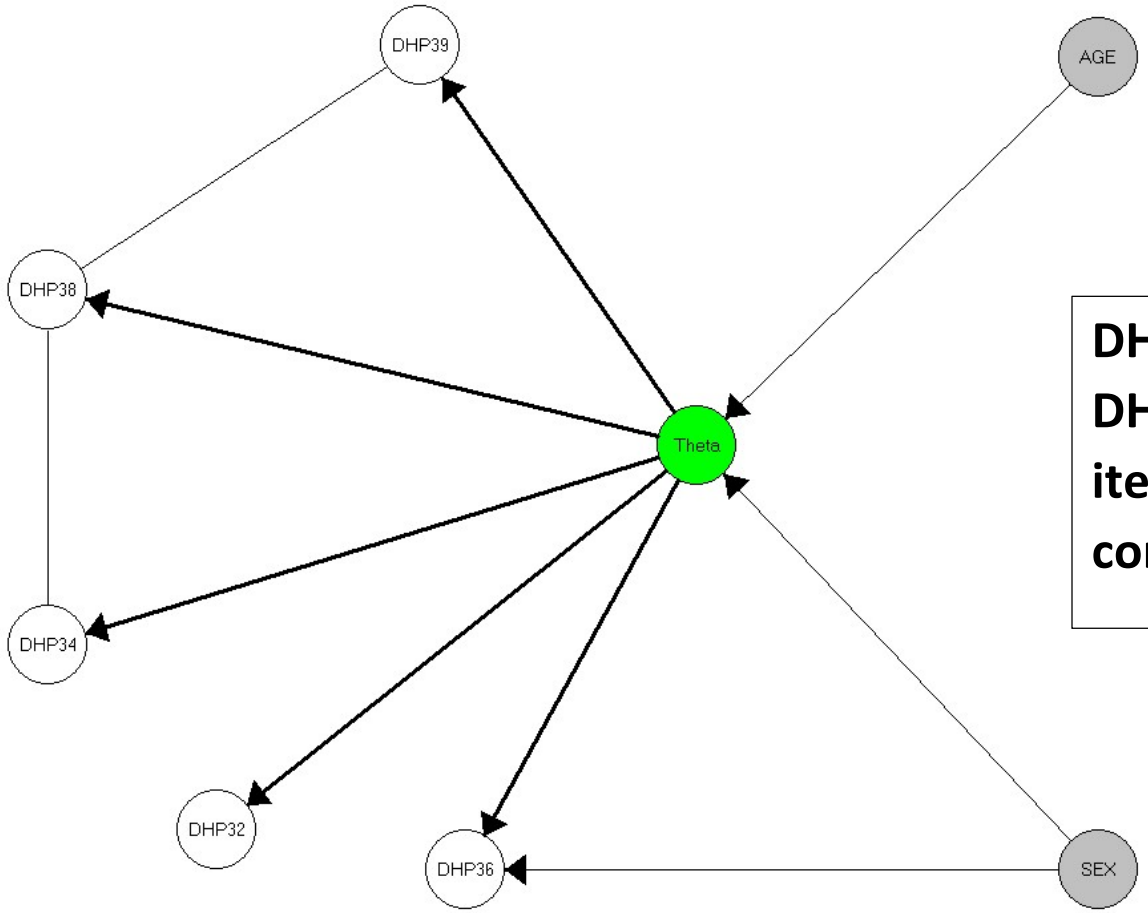
Graphical log-linear Rasch models (GLLRMs) with local dependence and DIF

$$\Pr(A=a,B=b,C=c,D=d,E=e \mid \theta,F=f,G=g) = \frac{\exp\left(r\theta+\sigma_a^A+\sigma_b^B+\sigma_c^C+\sigma_d^D+\sigma_e^E+\lambda_{bd}^{BD}+\lambda_{de}^{DE}+\varphi_{cg}^{CG} \mid \theta, F=f,G=g\right)}{H(\theta)}$$

DIGRAM fits GLLRMs with two-factor interactions between items and exogenous variables

The general class of log-linear Rasch models include higher order interactions

IRT and Rasch graphs are also defined for GLLRMs

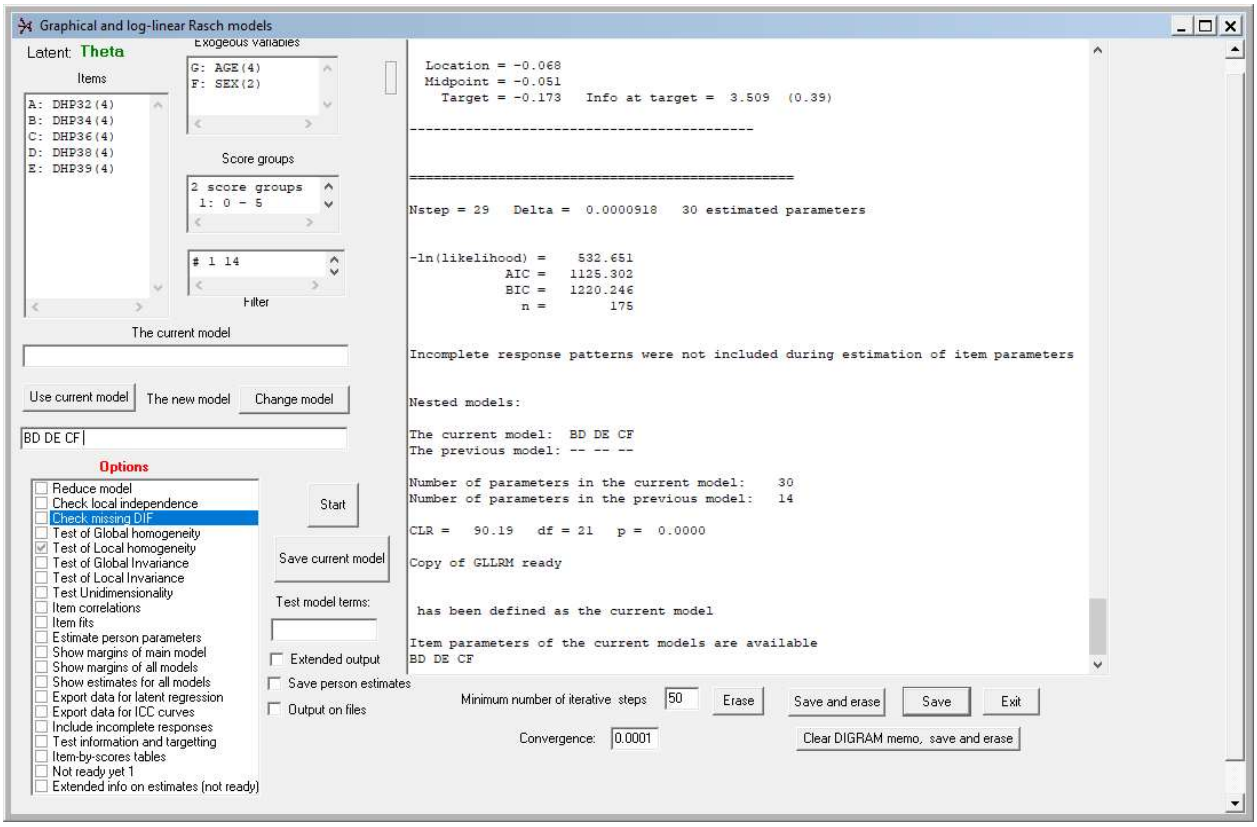


DHP34, DHP38 and DHP39 define an item component with connected items

The subscore over items in components has the same distribution as a polytomous Rasch item

The analysis

The current model is a Rasch model, but we define a new model by adding interactions to the "New model" field and click start to estimate the parameters



Calculate CLR tests of homogeneity and invariance

	CLR	df	p
scoregroups	19.2	30	0.936
F: AGE	99.9	90	0.223
G: SEX	25.2	24	0.394

Calculate item fit statistics

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| Item restscore association   |
|                              |
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```

Item		Item-restscore gamma observed	expected	sd	p
A -	DHP32	0.296	0.325	0.073	0.69862
B -	DHP34	0.494	0.497	0.058	0.96664
C -	DHP36	0.335	0.307	0.069	0.68380
D -	DHP38	0.681	0.658	0.056	0.68407
E -	DHP39	0.514	0.518	0.068	0.95517

Critical levels adjusted by the Benjamini-Hochberg procedure:
* < 5 % FDR, ** < 1 % FDR, *** = FDR < 0.1 % FDR

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| Component-restscore gamma coefficients |
|                                        |
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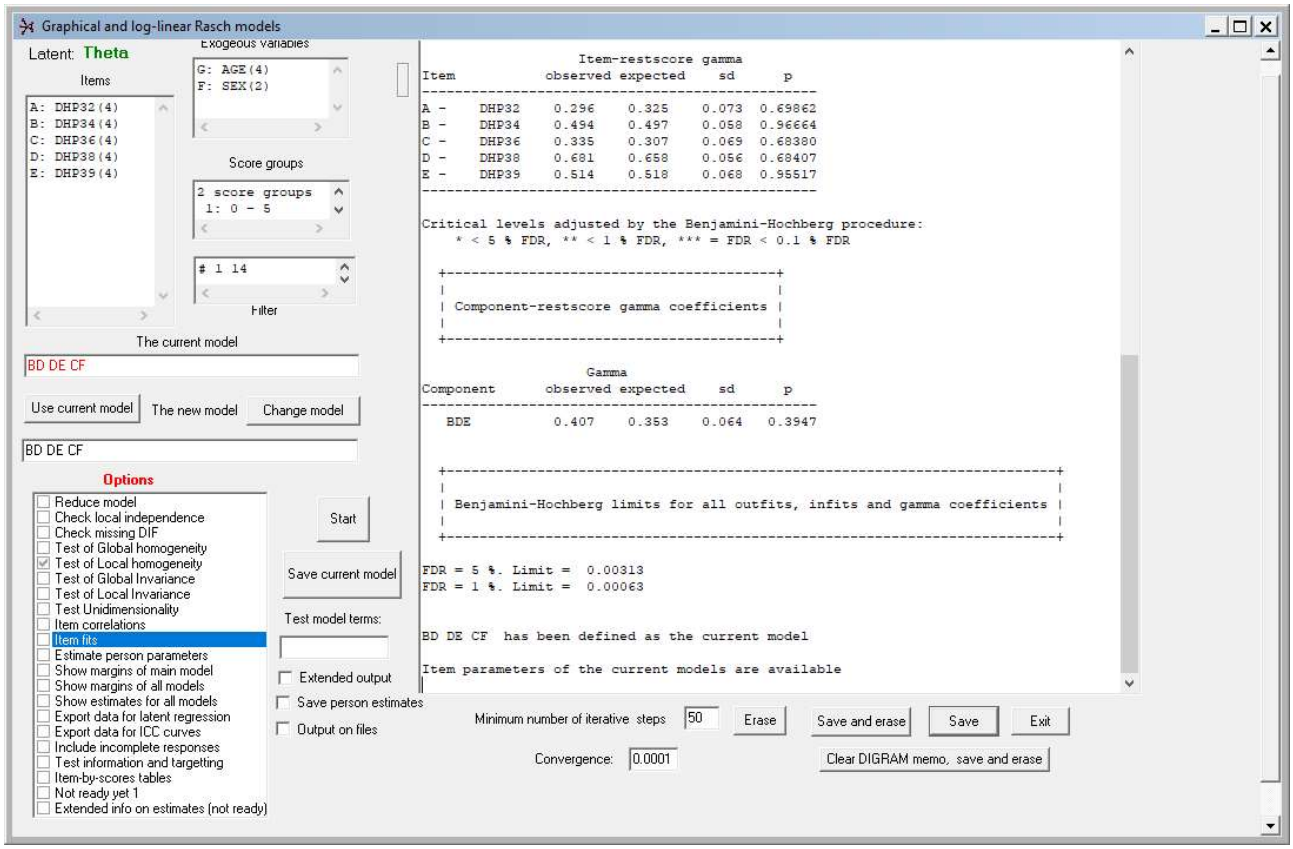
Component	Gamma observed	expected	sd	p
BDE	0.407	0.353	0.064	0.3947

No evidence against the new model turned up. We therefor select this model as the current model by clicking “Change”



GLLRM modelling options

Reduce model to test that we need all interactions

Check local independence to test whether there is more LD

Check missing DIF to test whether there is more DIF

Test model terms to test specific model terms

Since there is no additional evidence against the model, we accept the current model and save it on a DIGRAM command file so that it will be easy to redefine the model.

The DHP parameters

Multiplicative score parameters				
	item	0	1	2	3
A:	DHP32	1.000	1.750	0.741	0.257
B:	DHP34	1.000	0.724	0.662	0.827
C:	DHP36	1.000	1.863	4.460	2.103
D:	DHP38	1.000	4.031	1.662	1.707
E:	DHP39	1.000	3.792	0.918	1.312

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|                                        |
| Local dependence:  DHP34(B) & DHP38(D) |
|                                        |
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```

	B			
D	0	1	2	3
0	1.000	1.000	1.000	1.000
1	0.116	0.213	0.673	1.000
2	0.000	0.000	0.000	1.000
3	0.253	0.395	1.203	1.000

Standardized Gamma = 0.556 (G2OR = 3.50)

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|                                        |
| Local dependence:  DHP38(D) & DHP39(E) |
|                                        |
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```

	D			
E	0	1	2	3
0	1.000	0.227	0.000	0.000
1	1.000	1.856	0.401	0.141
2	1.000	1.080	5.269	0.290
3	1.000	1.000	1.000	1.000

Standardized Gamma = 0.445 (G2OR = 2.60)

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   |                          |
   | DIF:  DHP36(C) & SEX(G)  |
   |                          |
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```

G		C 0	1	2	3
1	Male	1.000	1.000	1.000	1.000
2	Female	1.000	0.326	0.320	0.252

Standardized Gamma = -0.350 (G2OR = 0.48)

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|                              |
| PCM thresholds and locations |
|                              |
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|                            |
| Locally independent items  |
|                            |
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```

	item	1	2	3	Location
A:	DHP32	-0.559	0.860	1.059	0.453
C:	DHP36 * DIF item *				
G =	Female	-0.622 >	-0.873	0.752	-0.248
G =	Male	0.498 >	-0.854	0.990	0.211

```
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|                                                     |
| Item components defined by local response dependence |
|                                                     |
+-----------------------------------------------------+
```

B-DHP34 & D-DHP38 & E-DHP39 Component scores from 0 to 9

Thresholds:

-1.531 -0.518 -0.308 > -0.617 -0.361 1.006 > -0.219 1.171 > 0.761

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|                            |
| Item and category effects  |
|                            |
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```

item		0	1	2	3	Item effect
A:	DHP32	0.000	1.013	0.606	0.000	-0.453
B:	DHP34	0.000	-0.260	-0.286	0.000	-0.063
C:	DHP36 * DIF item *					
F =	Male	0.000	0.375	1.000	-0.000	0.248
F =	Female	0.000	-0.286	0.779	0.000	-0.211
D:	DHP38	0.000	1.216	0.151	-0.000	0.178
E:	DHP39	0.000	1.242	-0.266	0.000	0.090
	---- MICE effects ----					
A:	DHP32	1.000	2.753	1.833	1.000	0.636
B:	DHP34	1.000	0.771	0.751	1.000	0.939
C:	DHP36 * DIF item *					
F =	Male	1.000	1.454	2.717	1.000	1.281
F =	Female	1.000	0.751	2.179	1.000	0.809
D:	DHP38	1.000	3.373	1.163	1.000	1.195
E:	DHP39	1.000	3.464	0.766	1.000	1.095

```
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  |                       |
  | Midpoints and targets |
  |                       |
  +-----------------------+

Item target = the person parameter where item information is maximized.
Midpoint    = the person parameter where the expected score = max score/2

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  |                           |
  | Locally independent items |
  |                           |
  +---------------------------+

A -    DHP32         Midpoint =  0.534  Target =  0.697  Info =  0.873  (0.29)
C -    DHP36
     G =   Female    Midpoint = -0.361  Target = -0.556  Info =  0.928  (0.31)
     G =   Male      Midpoint =  0.075  Target = -0.090  Info =  1.108  (0.37)

  +-------------------------------------------------------------+
  |                                                             |
  | Info on item components defined by local response dependence |
  |                                                             |
  +-------------------------------------------------------------+


BDE: B-DHP34 & D-DHP38 & E-DHP39

Component scores from 0 - 9.   Expected score at midpoint =  4.5

  Location = -0.068
  Midpoint = -0.051
    Target = -0.173   Info at target =  3.509  (0.39)
```
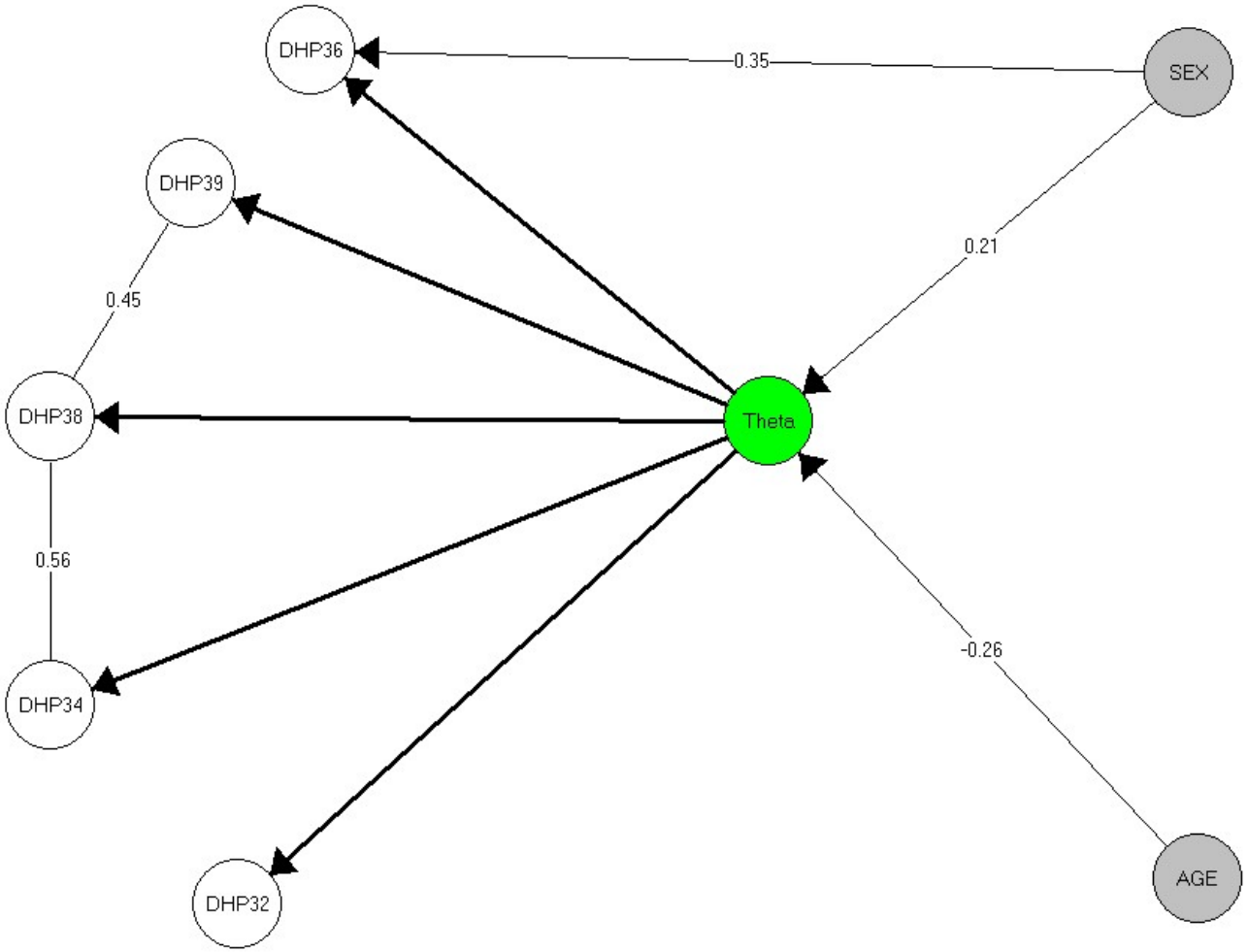

The IRT graph



Remaining issues

GRLLM analysis in practice

Item screening to define an initial GLLRM followed by log-linear model search

Assessment of person fit

Estimation of person parameters

Dealing with DIF by models with DIF

Assessment of measurement quality

CONF08 demo